Non-renormalization of the ‘chiral anomaly’ in interacting lattice Weyl semimetals

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Based on a joint work with
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Outline

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Overview

The Dirac equation describes relativistic electrons

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

and, if $m = 0$, is equivalent to two Weyl equations

$$\sigma^\mu \partial_\mu \psi_+ = 0, \qquad \bar{\sigma}^\mu \partial_\mu \psi_- = 0$$

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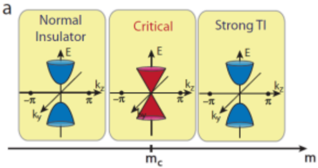
Recent: 3D Weyl semimetals with point-like Fermi surface.



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The $\frac{1}{2\pi^2}$ is the analogue of the Adler-Bell-Jackiw anomaly.

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We consider a class of lattice Weyl semimetals with short range interactions, and prove universality of the ABJ anomaly.

Mechanism: very different from Adler-Bardeen proof. Our ingredients: rigorous RG, regularity of current correlations, lattice Ward Identities. Important fact: short range interactions are irrelevant in the IR.

Outline

1. Overview
2. The chiral anomaly in QED_4
3. Lattice Weyl semimetals & Main results
4. Sketch of the proof

Massless Dirac fermions, gauge symmetries

Consider massless 4D Dirac fermions in a background field:

$$\mathcal{L}(\psi, A) = \bar{\psi}\gamma_\mu(i\partial_\mu - A_\mu)\psi$$

where $\bar{\psi} = \psi^\dagger\gamma_0$ and γ_μ are Euclidean Gamma matrices:

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu,\nu}, \quad \text{e.g. :} \quad \gamma_0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma_j = \begin{pmatrix} 0 & i\sigma_j \\ -i\sigma_j & 0 \end{pmatrix}.$$

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$\mathcal{L}$ covariant under local $U(1)$ gauge transformation:

$$\psi_{\mathbf{x}} \to e^{-i\alpha(\mathbf{x})}\psi_{\mathbf{x}}\ , \quad \psi^\dagger_{\mathbf{x}} \to \psi^\dagger_{\mathbf{x}} e^{+i\alpha(\mathbf{x})}, \quad A_{\mu,\mathbf{x}} \to A_{\mu,\mathbf{x}} + \partial_\mu\alpha(\mathbf{x}).$$

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$\mathcal{L}$ invariant under global axial $U(1)$ gauge transformation:

$$\psi_{\mathbf{x}} \to e^{i\gamma_5\alpha^5}\psi_{\mathbf{x}}, \qquad \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}.$$

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Axial symmetry: same as $\psi_{\mathbf{x},\omega} \to e^{-i\omega\alpha^5}\psi_{\mathbf{x},\omega}$, with $\omega = \pm$:

$$\begin{pmatrix} \psi_+ \\ 0 \end{pmatrix} = \frac{1+\gamma_5}{2}\psi, \quad \begin{pmatrix} 0 \\ \psi_- \end{pmatrix} = \frac{1-\gamma_5}{2}\psi, \quad \text{i.e.,} \quad \psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}.$$

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Axial symm. can be promoted to local $U(1)$, by adding to $\mathcal{L}$ an auxiliary term $-A^5_\mu\bar{\psi}\gamma_\mu\gamma_5\psi$, and letting

$$\psi_{\mathbf{x}} \to e^{-i\gamma^5\alpha^5(\mathbf{x})}\psi_{\mathbf{x}}\,, \quad \psi^\dagger_{\mathbf{x}} \to \psi^\dagger_{\mathbf{x}} e^{+i\gamma^5\alpha^5(\mathbf{x})}, \quad A^5_{\mu,\mathbf{x}} \to A^5_{\mu,\mathbf{x}} + \partial_\mu\alpha^5(\mathbf{x})$$

Conserved currents: classical and quantum

Classically, Noether's theorem $\Rightarrow$ $\boxed{\partial_\mu j_\mu = 0}$ and $\boxed{\partial_\mu j^5_\mu = 0}$ with $j_\mu = \bar{\psi}\gamma_\mu\psi$ and $j^5_\mu = \bar{\psi}\gamma^5\gamma_\mu\psi$, states conservation of the total and axial charges:

$$j_0 = \psi^\dagger\psi = \sum_{\omega=\pm}\psi^\dagger_\omega\psi_\omega, \qquad j^5_0 = \psi^\dagger\gamma_5\psi = \sum_{\omega=\pm}\omega\psi^\dagger_\omega\psi_\omega.$$

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These conservation laws might be broken in quantum theory, due to UV regularization.

$$e^{W(\mathbf{A},\mathbf{A}^5)} \propto \int D\psi e^{-i(\bar{\psi},\not{\partial}\psi)+(A_\mu,j_\mu)+(A^5_\mu,j^5_\mu)}.$$

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Formally, $W(\mathbf{A},\mathbf{A}^5) = W(\mathbf{A}+\boldsymbol{\partial}\alpha, \mathbf{A}^5+\boldsymbol{\partial}\alpha^5)$, from which

$$\langle\partial_\mu j_\mu\rangle_{\mathbf{A}} = 0, \qquad \langle\partial_\mu j^5_\mu\rangle_{\mathbf{A}} = 0,$$

where $$\langle O(\psi)\rangle_{\mathbf{A}} = \frac{\int D\psi e^{-i(\bar{\psi},\not{\partial}\psi)+(A_\mu,j_\mu)}O(\psi)}{\int D\psi e^{-i(\bar{\psi},\not{\partial}\psi)+(A_\mu,j_\mu)}}.$$

Loop cancellation and UV divergences

Note: $\langle \partial_\mu \jmath^\sharp_\mu \rangle_{\mathbf{A}} = 0$ is the same as [letting $\mathbf{p} = \mathbf{p}_1 + \cdots + \mathbf{p}_n$]

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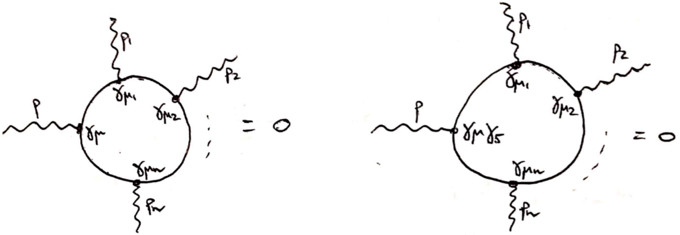
That is, all loop diagrams cancel:



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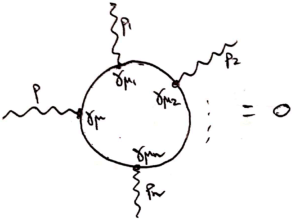
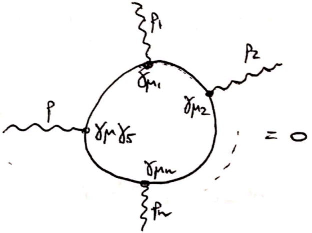




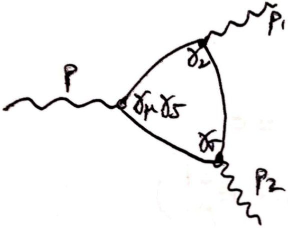
However, the loop diagrams with $n \leq 3$ are UV divergent! We need an UV regularization (to be eventually removed) in order to give the diagrams and to the cancellations a meaning.

The axial anomaly

Fact: there is no way to add an UV regularization preserving both the vectorial and axial current conservations. If we choose to preserve the vectorial $U(1)$ gauge symmetry, then

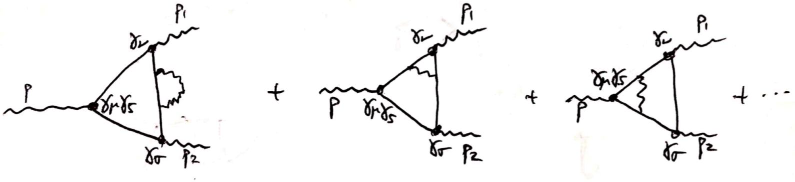
$$\langle \partial_\mu j^5_\mu \rangle_{\mathbf{A}} = -\frac{i}{2\pi^2} \varepsilon_{\alpha\beta\nu\sigma} \partial_\alpha A_\nu \partial_\beta A_\sigma \ .$$

$\frac{1}{2\pi^2}$ is the ABJ anomaly, determined by the triangle graph:



Radiative corrections, Adler-Bardeen theorem

What if we add interactions, i.e., coupling with dynamical e.m. field? Is the triangle graph dressed by radiative corrections?



Adler-Bardeen theorem: NO! All possible dressings of the triangle cancel exactly. Required: specific UV regularization, exact relativistic invariance of the fermionic propagator.

[Deep consequences in QED and Standard Model: exact decay rate of $\pi^0 \to \gamma\gamma$, constraint on the number of lepton/quark families.]

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where $\alpha(k) = t_3(2 + \zeta - \cos k_1 - \cos k_2 - \cos k_3)$ and $\beta(k) = t_1 \sin k_1 - it_2 \sin k_2$, with $t_1, t_2, t_3 > 0$ hopping parameters, and $-1 < \zeta < 1$ tunes location of Weyl nodes.

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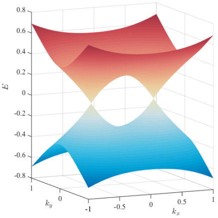
Energy bands touch with conical singularity at $k = p_F^\pm$, with $p_F^\pm = (0, 0, \pm \arccos \zeta)$: Weyl nodes merge as $\zeta \to 1^-$



Symmetries

The Hamiltonian breaks time-reversal symmetry:

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$$\hat{H}^0(k) = \hat{H}^0\big((k_1, k_2, -k_3)\big) \quad \text{and} \quad \hat{H}^0(k) = -\sigma_1 \hat{H}^0\big((-k_1, k_2, k_3)\big)\sigma_1$$

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The WSM phase of $\hat{H}^0(k)$ appears as a transitional state between a NI ($\zeta < -1$) and a TI ($\zeta > 1$).

Effective Weyl Hamiltonian and two-point function

In the vicinity of the Weyl nodes $p_F^\omega = (0, 0, \omega \arccos \zeta)$,

$$\hat{H}^0(k) \simeq v_1^0 k_1' \sigma_1 + v_2^0 k_2' \sigma_2 + \omega v_3^0 \sigma_3 k_3',$$

where $(v_1^0, v_2^0, v_3^0) = (t_1, t_2, t_3\sqrt{1-\zeta^2})$ are the free Fermi velocities, ω is the chirality index, and $k' = k - p_F^\omega$.

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If $\quad H_0 = \int_{[-\pi,\pi]^3} \frac{dk}{(2\pi)^3} \hat{a}_k^+ \hat{H}^0(k) \hat{a}_k^- \quad$ and $\quad \langle \cdot \rangle_0 = \lim_{\beta, L \to \infty} \frac{\mathrm{Tr}(e^{-\beta H_0} \cdot)}{\mathrm{Tr}\, e^{-\beta H_0}},$

then the two-point function reads:

$$\langle a_x^- a_y^+ \rangle_0 = \int \frac{dk_0\, d^3k}{(2\pi)^4} (-ik_0 + \hat{H}_0(k))^{-1} e^{-ik\cdot(x-y)}$$

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where $(\sigma_\mu^\omega) = (-i\mathbb{1}, \sigma_1, \sigma_2, \omega\sigma_3)$, and $v_0^0 \equiv 1$, $k_0' \equiv k_0$.

Lattice interacting model and lattice chiral charge

We consider an interacting version of the model:

$$H = H_0 + \lambda V_0 + \nu N_3$$

where V_0 is a short-range density-density interaction, $N_3 = \int \frac{dk}{(2\pi)^3} \hat{\psi}^+_k \sigma_3 \hat{\psi}^-_k$, and ν is used to fix the location of p^ω_F.

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We let $\langle \cdot \rangle = \lim_{\beta, L \to \infty} \mathrm{Tr}(e^{-\beta H} \cdot)/\mathrm{Tr}\, e^{-\beta H}$. We are interested in the response of $N^5 = \sum_x n_x^5$ to an external e.m. field, where

$$n_x^5 = \frac{Z^5_{\mathrm{bare}}}{4} \sum_{\delta = \pm} (i a^+_x a^-_{x+\delta e_3} - i a^+_{x+\delta e_3} a^-_x).$$

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We let $\langle \cdot \rangle = \lim_{\beta, L \to \infty} \mathrm{Tr}(e^{-\beta H} \cdot)/\mathrm{Tr}\, e^{-\beta H}$. We are interested in the response of $N^5 = \sum_x n^5_x$ to an external e.m. field, where

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Lattice interacting model and lattice chiral charge

We consider an interacting version of the model:

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$Z^5_{\rm bare}$ fixed by imposing: dressed chiral charge $\equiv$ electric charge.

Coupling to an external e.m. field

Gauge invariant coupling to an external e.m. field: any hopping $t_{x,y} a^+_{i,x} a^-_{j,y}$ is modified into (Peierl's substitution):

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Note: the interaction is gauge invariant.

Also the chiral density is coupled to the e.m. field:

$$n^5_x(A) = \frac{Z^5_{\rm bare}}{4} \sum_{\delta=\pm} (i a^+_x a^-_{x+e_3} e^{i \int_0^1 A_3(x+s\delta e_3) ds} + H.c.)$$

Quadratic response coefficient of the chiral density

External e.m. potential: $A_x(t) = e^{\eta t} A_x$ for $t \leq 0$, with $\eta > 0$ small (adiabatic switching), A_x slowly varying in space.

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By formally expanding in A, for A small with zero average:

$$\lim_{\beta, L \to \infty} L^{-3} \langle N^5(A(t)) \rangle_{\beta,L;t} = \frac{i}{2} \int \frac{dp}{(2\pi)^3} \big(\hat{A}_p(t), \hat{\Pi}^5_0(\eta, p) \hat{A}_{-p}(t) \big) + O(A^3).$$

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It is the analogue of the dressed triangle graph in QED_4.

Main result

Theorem (G., Mastropietro, Porta 2020)

For $|\lambda|$ small enough and any $\zeta \in (-1, 1)$, there exists $\nu = \nu(\lambda) = O(\lambda)$ s.t. interacting two-point function is singular at $p_F^{\pm}$, $Z^5_{\rm bare} = Z^5_{\rm bare}(\lambda) = 1 + O(\lambda)$ such that the chiral dressed charge equals the electric charge.

Fixing ν and $Z^5_{\rm bare}$ this way, for small enough η, p:

$$\hat{\Pi}^5_{0,i,j}(\eta, p) = -\frac{1}{2\pi^2} \sum_{l=1}^{3} \varepsilon_{ijl}\, p_l + \hat{R}^5(\eta, p),$$

where ε_{ijl} is the Levi-Civita symbol and, for any $\theta \in (0, 1)$:

$$|\hat{R}^5(\eta, p)| = O([\max\{\eta, |p|\}]^{1+\theta})$$

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we find that:

$$\partial_t\langle n^5(A(t))\rangle_t \simeq \frac{1}{2\pi^2}\int dx\, E_x(t)\cdot B_x(t),$$

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This equation has measurable implications (negative longitudinal magnetoresistance) for real Weyl SM.

Outline

Asymptotic freedom of the infrared theory

Ground state correlations constructed via a rigorous RG, analogous to the one used by G-Mastropietro for the half-filled Hubbard model on the hexagonal lattice.

IR theory of WSM, even if massless, very well behaved: quartic interaction is irrelevant $\Rightarrow$ scales to zero at large distances (scaling dimension of $\psi^n A^m$ kernels: $D = 4 - \frac{3}{2}n - m$).

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Large distance decay of two-point and vertex functions

$$\hat{S}_2(\mathbf{k}) := \langle \hat{a}^-_{\mathbf{k}} \hat{a}^+_{\mathbf{k}} \rangle \quad \text{and} \quad \hat{\Gamma}_0(\mathbf{k}, \mathbf{p}) := \langle \hat{n}_{\mathbf{p}}; \hat{a}^-_{\mathbf{k}+\mathbf{p}} \hat{a}^+_{\mathbf{k}} \rangle$$

same as for non-interacting model, up to finite multiplicative renormalization and finite dressing of the Fermi velocities.

Dressed two-point and vertex function

Interacting two-point function. For $\mathbf{k}'$ small:

$$\hat{S}_2(\mathbf{k}' + \mathbf{p}_F^\omega) \simeq \frac{1}{Z}\Big(\sum_{\mu=0}^{3} \sigma_\mu^\omega v_\mu k'_\mu\Big)^{-1},$$

with $Z = Z(\lambda) = 1 + O(\lambda)$ and $v_\mu = v_\mu^0(1 + O(\lambda))$, $v_0 \equiv 1$.

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$$\hat{\Gamma}_0(\mathbf{k}' + \mathbf{p}_F^\omega, \mathbf{p}) \simeq Z_0 \hat{S}_2(\mathbf{k}' + \mathbf{p} + \mathbf{p}_F^\omega)\hat{S}_2(\mathbf{k}' + \mathbf{p}_F^\omega).$$

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A Ward Identity guarantees that $\boxed{Z_0 = Z}$

Physical meaning: dressed electric charge $\equiv$ bare charge.

Dressed chiral vertex

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with $\tilde{Z} = \tilde{Z}(\lambda) = 1 + O(\lambda)$. NOTE: no chiral gauge symmetry $\Rightarrow$ no a priori relation between relating $Z^5_0 := \tilde{Z} Z^5_{\rm bare}$ and Z.

Criterium for fixing $Z^5_{\rm bare}$ ('same' used by Adler in QED$_4$):

$$Z^5_0 = \tilde{Z}\, Z^5_{\rm bare} \equiv Z$$

which means: dressed chiral charge $\equiv$ electric charge.

IR decomposition of quadratic response

Vertex functions for (non-)chiral density can be generalized to (non-)chiral lattice currents $\hat{J}_{\mu,\mathbf{p}}$ and $\hat{J}^5_{\mu,\mathbf{p}}$, $\mu = 0, 1, 2, 3$.

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Bare constants $Z^5_{\mu,\mathrm{bare}}$ fixed s.t. $Z^5_\mu = Z_\mu$.

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Quadratic response of $\hat{J}^5_{\mu,\mathbf{p}}$ to external e.m. field: $\hat{\Pi}^5_{\mu,\nu,\sigma}(\mathbf{p}_1, \mathbf{p}_2)$

Asymptotic freedom in IR + Ward Identities + $(Z^5_\mu = Z_\mu)$ $\Rightarrow$

$$\boxed{\hat{\Pi}^5_{\mu,\nu,\sigma}(\mathbf{p}_1, \mathbf{p}_2) = \frac{v_\mu v_\nu v_\sigma}{v_1 v_2 v_3} I_{\mu,\nu,\sigma}(\overline{\mathbf{p}}_1, \overline{\mathbf{p}}_2) + H^5_{\mu,\nu,\sigma}(\mathbf{p}_1, \mathbf{p}_2)} \qquad (*)$$

where:

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where: $I_{\mu,\nu,\sigma}(\mathbf{p}_1, \mathbf{p}_2)$ is the relativistic chiral triangle graph with momentum cutoff, $\bar{p}_{i,\mu} = v_\mu p_{i,\mu}$ and the *subdominant* term $H^5_{\mu,\nu,\sigma}$ is $C^{1+\theta}$ in a neighborhood of $(\mathbf{0}, \mathbf{0})$ (while $I_{\mu,\nu,\sigma}$ has discontinuous derivatives at the origin).

The relativistic chiral triangle graph

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where $\overline{\mathbf{p}}_i = (p_{i,0}, v_1 p_{i,1}, v_2 p_{i,2}, v_3 p_{i,3})$, $H^5_{\mu,\nu,\sigma} \in C^{1+\theta}$ and

$$I_{\mu,\nu,\sigma}(\mathbf{p}_1,\mathbf{p}_2) = \int \frac{d\mathbf{k}}{(2\pi)^4} \mathrm{Tr}\Big\{ \frac{\chi(\mathbf{k})}{\not{\mathbf{k}}} \gamma_\mu \gamma_5 \frac{\chi(\mathbf{k}+\mathbf{p}_1)}{\not{\mathbf{k}}+\not{\mathbf{p}}_1} \gamma_\nu \frac{\chi(\mathbf{k}+\mathbf{p}_2)}{\not{\mathbf{k}}+\not{\mathbf{p}}_2} \gamma_\sigma \Big\} +$$

$$+ \left[(\nu,\mathbf{p}_1) \leftrightarrow (\sigma,\mathbf{p}_2)\right].$$

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$+ [(\nu,\mathbf{p}_1) \leftrightarrow (\sigma,\mathbf{p}_2)]$. A computation shows that

$$\sum_{\mu=0}^{3} (p_{1,\mu} + p_{2,\mu}) I_{\mu,\nu,\sigma}(\mathbf{p}_1,\mathbf{p}_2) = \frac{1}{6\pi^2} p_{1,\alpha} p_{2,\beta} \varepsilon_{\alpha\beta\nu\sigma} + h.o.,$$

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Lattice Ward identities

We now use $\sum_\nu p_{1,\nu}\hat{\Pi}^5_{\mu,\nu,\sigma}(\mathbf{p}_1,\mathbf{p}_2) = 0$, which implies, together with $(*)$ and the $C^{1+\theta}$ differentiability of $H^5_{\mu,\nu,\sigma}$:

$$\frac{p_{1,\alpha}p_{2,\beta}\varepsilon_{\alpha\beta\mu\sigma}}{6\pi^2}+p_{1,\nu}\Big(H^5_{\mu,\nu,\sigma}(\mathbf{0},\mathbf{0})+\sum_{j=1,2}p_{j,\alpha}\frac{\partial H^5_{\mu,\nu,\sigma}}{\partial p_{j,\alpha}}(\mathbf{0},\mathbf{0})\Big) = O(P^{2+\theta}),$$

with $P = \max\{|\mathbf{p}_1|,|\mathbf{p}_2|\}$.

Lattice Ward identities

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First term: we computed it explicitly. Second term: use (**).

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– Open problems:

1. Effects of disorder?
2. Coupling to a dynamical e.m. field: rigorous construction of infrared QED_4 [without photon mass counterterms]? Dynamical restoration of Lorentz invariance in the IR? Non-renormalization of the chiral anomaly?

Thank you!