

MATHEMATICAL CHALLENGES IN QUANTUM MECHANICS

QUANTUM HALL EFFECT: TOPOLOGY IN THE LAB

Domenico Monaco, Sapienza Uni Roma 1

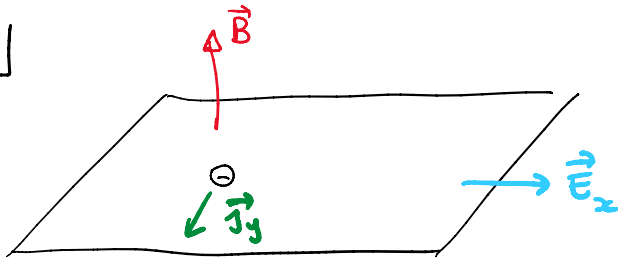
OUTLINE

- Phenomenology : The bulk & edge picture of the QHE
The bulk-edge correspondence (BEC)
- Mathematical results: Kubo & Středa formulas for bulk conductivity
Edge conductivity and BEC
- Snapshots from the proof : from Kubo to Chern

Credits: Avron, Seiler, Simon, Aizenman, Graf, Bellissard, Schulz-Baldes, Kellendonk, Spohn, Panati, Giuliani, Mastropietro, Porta, Hastings, Michalakis, Bachmann, De Roeck, Fraas, ... & Teufel

EFFETTO HALL QUANTISTICO (QHE)

BULK



2D e^- -gas

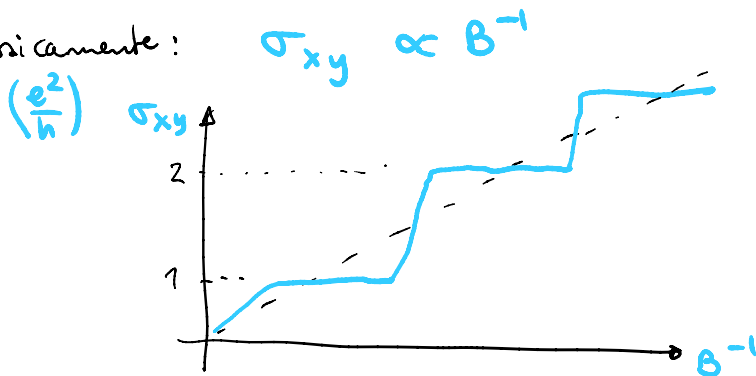
Isolante \rightarrow corrente trasversa

Legge di Ohm:

$$j_y = \sigma_{xy} E_x$$

Conduttività Hall

Classicamente:

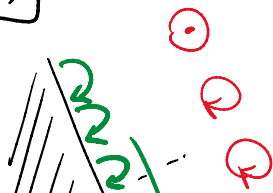


QHE : PLATEAUX

$$\sigma_{xy} = n \frac{e^2}{h} = \frac{1}{2\pi} n$$

$n \in \mathbb{Z}$

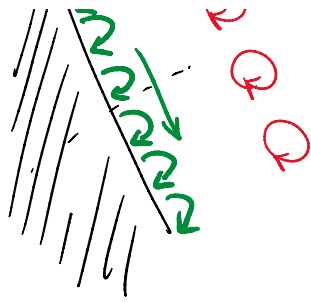
BORDO (EDGE)



Troncamento del cristallo

\rightarrow correnti di bordo





→ correnti di bordo

$$\sigma_{\text{edge}} = \sigma_{\text{bulk}}$$

Corrispondenza bulk-edge

MATEMATICA

① Bulk: trasporto, risposta lineare, invarianti topologici

H_0 : Schrödinger con campo magnetico uniforme & potenziale periodico

$$H_0 = \frac{1}{2} (-i\vec{\nabla} - \vec{A}(\vec{x}))^2 + V(\vec{x})$$

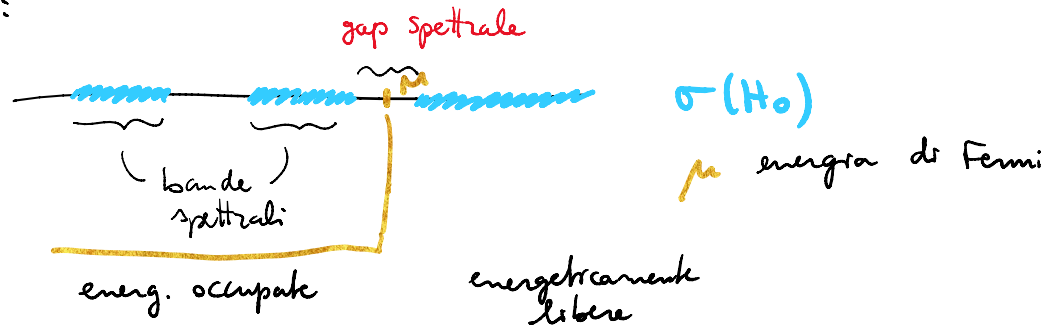
$$\vec{A}(x,y) = \frac{B}{2} (-y, x)$$

$$V(x-R) = V(x) \quad R \in \mathbb{Z}^2$$

no commuta con traslazioni magnetiche: $[H_0, T_R] = 0$

$$(T_R \psi)(x) = e^{iA(x) \cdot R} \psi(x-R) \quad R \in \mathbb{Z}^2$$

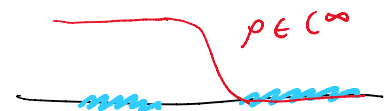
In generale:



Proiettore di Fermi: stato di equilibrio a $T=0$ del sistema

$$\Pi_0 = \mathbb{1}_{(-\infty, \mu)}(H_0)$$

$$= \rho(H_0)$$



Induce il campo elettrico esterno: $H_E = H_0 - EX$

altimenti $H_E(t) = H_0 - f(t) EX$

$$f(t) = e^{\eta t} \chi(t \leq 0)$$

Interessa : misurare la corrente di risposta: $J_Y = i [H, Y]$
 $= i [H_0, Y]$

→ $\langle J_Y \rangle_{\Pi_E}$ (almeno al 1° ordine in E)

$\langle \cdot \rangle = ?$ Sistema esteso : $\text{Tr} (J_Y \Pi_E) = \infty$ per
 "periodicità"

Traccia per unità di volume

$$\tau(A) = \lim_{L \rightarrow \infty} \frac{1}{L^d} \text{Tr} (\mathbb{1}_L A \mathbb{1}_L)$$

$$= \lim_{L \rightarrow \infty} \frac{1}{L^d} \int_{[-\frac{L}{2}, \frac{L}{2}]^d} A(x; x) dx$$

A periodico : $\tau(A) = \int_{[-\frac{1}{2}, \frac{1}{2}]^d} A(x; x) dx$

TEOREMA. FORMULA DI KUBO : nella risposta lineare

$$\langle J_Y \rangle_{\Pi_E} = \tau (J_Y \Pi_E) = \sigma_{xy} E \quad (+ \mathcal{O}(E^2))$$

dove

$$\sigma_{xy} = i \tau (\Pi_0 [[\Pi_0, X], [\Pi_0, Y]])$$

Marker di Chern / Numero di Chern: $\sigma_{xy} \in \frac{1}{2\pi} \mathbb{Z}$

- ① Geom. differenziale : no. di Chern per il fibrato vettoriale associato agli stati energeticamente occupati sopra il loro dei momenti cristallini
- ② NCG : indice di Fredholm
- ③ Indice di una coppia di proiettori
- ④ ...

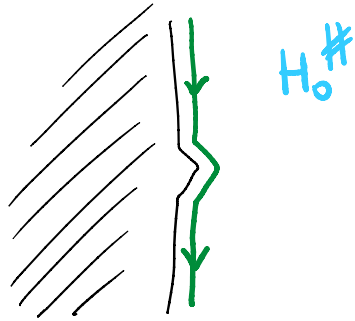
FORMULA DI STÄNEDA

IDOS : $\mathcal{I}(B) := \tau(\Pi_0(B))$

TEOREMA. (1) $\sigma_{xy} = \frac{d\gamma}{dB} = i \tau (\rho_0 [(\rho_0, x), (\rho_0, y)])$

(2) σ_{xy} e' costante in B : $\gamma(B) = \underbrace{c_0}_{\in \mathbb{Q}} + \underbrace{c_1}_{\in \mathbb{Z}} \frac{B}{2\pi}$

BOUND



$$\sigma_{\text{ess}}(H_0^\#) = \sigma_{\text{ess}}(H_0)$$

$\rightarrow \sigma_{\text{discre}}(H_0^\#)$: stati di bordo

$$\sigma_{\text{edge}} = \tau (\rho'(H_0) J_Y)$$

BEC : $\sigma_{\text{edge}} = \sigma_{xy} \in \frac{1}{2\pi} \mathbb{Z}$